

Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2

Branch: B. Tech (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$, then for any positive integer n, the value of $n(I_{n-1} + I_{n+1})$

is

(A) 1 (B) 2 (C) $\frac{\pi}{4}$ (D) π

b) Find the value of $\int_{-1}^1 \sin^{11} x dx$

(A) 10! (B) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{\pi}{2}$ (C) 0 (D) none of these

c) $\sqrt{4.5} = \underline{\hspace{2cm}}$

(A) $\frac{\sqrt{\pi}}{16}$ (B) $\frac{105\sqrt{\pi}}{16}$ (C) $\frac{5\sqrt{\pi}}{16}$ (D) none of these

d) If $B(x, 2) = \frac{1}{3}$, then the value of $x = \underline{\hspace{2cm}}$.

(A) 0 (B) 1 (C) 2 (D) none of these

e) $\int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{1}{4} \sin^2 \theta} d\theta$ is equal to

(A) $E\left(\frac{1}{2}\right)$ (B) $E\left(\frac{1}{4}\right)$ (C) $K\left(\frac{1}{2}\right)$ (D) $K\left(\frac{1}{4}\right)$

f) $\int_{-a}^a e^{-t^2} dt$ is equal to



(A) $\sqrt{\pi} \operatorname{erf}(a)$ (B) $\sqrt{\pi} \operatorname{erfc}(a)$ (C) $\frac{\sqrt{\pi}}{2} \operatorname{erf}(a)$ (D) $\frac{\sqrt{\pi}}{2} \operatorname{erfc}(a)$

- g) If the power of y are even, then the curve is symmetrical about
 (A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these
- h) The curve $y^2(a+x) = x^2(a-x)$ where $a > 0$ represent
 (A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
 (D) Folium of Descartes
- i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx dy$ is equal to
 (A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these
- j) The transformations $x+y=u, y=uv$ transform the area element $dy dx$ into $|J| du dv$, where $|J|$ is equal to
 (A) 1 (B) u (C) -1 (D) none of these
- k) The degree of the differential equation $3 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}$ is
 (A) 1 (B) 2 (C) 3 (D) 6
- l) If $\frac{dy}{dx} + \frac{1}{y\sqrt{1-x^2}} = 0$, then which of the following statements is true?
 (A) $y + \sin^{-1} x = 0$ (B) $y^2 + 2\sin^{-1} x = c$ (C) $x + \sin^{-1} y = c$ (D) $y = k$
- m) The series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \dots \infty$ is
 (A) convergent (B) divergent (C) finitely oscillating (D) infinitely oscillating
- n) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (A) $\log 2$ (B) zero (C) infinite (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Using reduction formula evaluate: $\int_0^1 x^6 \sin^{-1} x dx$ (5)

b) Prove that $\int_{-\infty}^{\infty} e^{-k^2 x^2} dx = \frac{\sqrt{\pi}}{k}$ (5)

c) Evaluate: $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z dx dy dz$ (4)

Q-3 Attempt all questions (14)



a) Show that $\int_0^2 x^4 (8-x^3)^{-\frac{1}{3}} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$ (5)

b) Solve: $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$ (5)

c) Discuss the convergence of $\sum \frac{\sqrt{n+1} - \sqrt{n}}{n}$. (4)

Q-4 Attempt all questions (14)

a) Change the order of integration in the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ and evaluate it. (5)

b) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \dots$ (5)

c) Evaluate: $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2} dx$ (4)

Q-5 Attempt all questions (14)

a) Solve: $x dy - y dx = x \sqrt{x^2 - y^2} dx$ (5)

b) By changing into polar co-ordinates, evaluate the integral (5)

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$$

c) Prove that $B(m, n)B(m+n, p)B(m+n+p, q) = \frac{\Gamma(m) \Gamma(n) \Gamma(p) \Gamma(q)}{\Gamma(m+n+p+q)}$ (4)

Q-6 Attempt all questions (14)

a) Show that $\sum_{n=1}^\infty \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ is (i) convergent if $p > 1$ and (ii) divergent if $p \leq 1$. (5)

b) Using reduction formula prove that $\int_0^1 \frac{x^6}{1+x^2} dx = \frac{13}{15} - \frac{\pi}{4}$. (5)

c) Solve: $(x^2 + y^2 - a^2)x dx + (x^2 - y^2 - b^2)y dy = 0$ (4)

Q-7 Attempt all questions (14)

a) Trace the curve $xy^2 = 4a^2(2a-x)$. (5)

b) Find the area of the loop of the curve $x^3 + y^3 = 3axy$. (5)

c) Evaluate: $\int_1^\infty \frac{dx}{\sqrt{x^4-1}}$ (4)

Q-8 Attempt all questions (14)

a) Prove that $erf_c(x) + erf_c(-x) = 2$. (5)

b) Find the asymptotes of the curve $y^3 - x^2(6-x) = 0$. (5)

c) Find the perimeter of the cardioids $r = a(1 + \cos \theta)$. (4)

